The Aeroelastic Similarity Hypothesis for Tree Structure

A Mechanical Design Principle for the Dissipation of Wind-Induced Motion

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Abstract

Measurements of the motions of Sitka spruce made as part of a wider investigation of the interaction between atmospheric turbulence and a conifer canopy revealed an unexpected relationship between the natural-frequency modes of the different structural elements of the tree. In particular, the first branch mode lay in the region of the second whole-tree mode, while higher branch and shoot modes suggested a continuing hierarchy of corresponding resonances. Sway tests and acceleration spectra also showed that the resonances were not sharp single frequencies but occupied finite bands associated with the natural variation in dimensions of nominally similar tree elements. Visual observations demonstrated mechanical coupling within the tree, while the measurements indicated important aeroelastic interactions between the bole, branches, shoots, twigs and needles. The tree responded to transient wind loading as a highly damped coupled structure. These observations led to a qualitative hypothetical mechanical design principle termed *aeroelastic similarity*. The principle proposes an organised equivalence between the modal regimes of successive tree elements, so that atmospheric energy entering one part of the structure can be transferred through mechanically coupled elements and redistributed through mechanical coupling and dissipated through the damping of the moving structure. The resulting tree acts as a distributed shock-absorbing system in which wind energy is partitioned among many modes, restricting the development of large resonant amplitudes that could exceed the elastic limits of individual elements. This paper presents the aeroelastic similarity hypothesis as developed from the experimental and theoretical work reported in the author's 1983 doctoral thesis.

1 Introduction

The response of a tree to turbulent wind is determined not only by the flexibility of its bole, but by the motions of a hierarchy of structural elements: branches, shoots, twigs and needles. Each of these elements possesses natural modes of vibration and each is mechanically connected to the other elements of the tree. The canopy is therefore a coupled aeroelastic structure exposed to a broad spectrum of atmospheric motion.

The work described here formed part of a doctoral investigation of the interactive motions of the atmospheric surface layer and a conifer canopy (Scannell, 1983). The atmospheric measurements, instrumentation and detailed turbulence results of that investigation are outside the scope of the present paper. Attention is restricted to one result arising from the measurements of tree motion: the apparent organisation of the resonant modes of the different structural levels of Sitka spruce and the mechanical interpretation that followed from it.

The original study combined natural-frequency estimates from sway tests, acceleration spectra measured during wind excitation, and theoretical modal calculations. Comparison of these results suggested a simple connection between the resonances of the separate canopy elements. Together with observations of strong mechanical coupling and rapid damping after wind excitation, this led to the proposition that the tree behaves as a large, well-structured shock absorber. A qualitative mechanical design principle, termed *aeroelastic similarity*, was proposed to describe this organisation.

2 Natural modes of the tree elements

Natural frequencies of the whole tree and of selected branches, shoots and twigs were estimated by controlled sway tests and from spectral peaks in acceleration records obtained under natural wind loading. The experimental results were compared with modal calculations based on measured flexibility coefficients and elemental masses.

For an undamped discretised element the natural modes may be written in the usual form

$$\left([K] - \omega^2 [M]\right) \mathbf{q} = 0, \tag{1}$$

where $[K]$ and $[M]$ are the stiffness and mass matrices, ω is natural angular frequency and $\mathbf{q}$ is the mode shape. In the thesis the experimentally determined flexibility matrix $[C] = [K]^{-1}$ was used, giving

$$\left([I] - \omega^2 [C][M]\right) \mathbf{q} = 0. \tag{2}$$

The flexibility coefficients were obtained by applying known loads at stations on sampled elements and measuring the resulting deflections.

For the first modes, the theoretical calculations overestimated the sway-test estimates by approximately 15% for branches and approximately 30% for shoots and twigs. The discrepancy was attributed principally to effects omitted when the elements were treated in relative isolation. In the actual tree, shoots, twigs and needles modify branch motion, while the bole, branches and smaller elements interact mechanically and aerodynamically. The measured motion is therefore that of an aeroelastic system rather than a collection of independent elastic cantilevers.

3 An organised equivalence of modes

Comparison of the whole-tree and branch spectra produced an unexpected result. Each identified branch mode was similar in frequency to the next higher whole-tree mode. If N_i denotes the ith whole-tree mode and B_i the ith branch mode, the observed relationship may be written schematically as

$$B_1 \simeq N_2, \qquad B_2 \simeq N_3, \qquad B_3 \simeq N_4. \tag{3}$$

The similarity was not predicted by the isolated-element theoretical calculations. Estimates obtained independently from sway tests and spectral peaks agreed on average to within approximately 10%, supporting the identification of the modes.

The shoot frequencies suggested that the relationship continued to the next structural level. In particular, the first shoot mode could be associated with the second branch and third whole-tree modes, giving

$$S_1 \simeq B_2 \simeq N_3, \tag{4}$$

and the higher estimates suggested

$$S_2 \simeq B_3 \simeq N_4. \tag{5}$$

The corresponding scheme was extended to the twig scale. The modal organisation proposed in the thesis is reproduced schematically in Table 1.

The scheme represents the proposed organisation of the modes; the experimental identification of the higher modes, particularly those of the smallest elements, was less certain than that of the lower whole-tree and branch modes.

Table 1: Scheme for the equivalent modes of the structural elements. The entries denote corresponding modal regimes rather than exact equality of individual natural frequencies.

Element	Modal group 1	Modal group 2	Modal group 3	Modal group 4
Whole tree	N_1	N_2	N_3	N_4
Branch		B_1	B_2	B_3
Shoot			S_1	S_2
Twig				T_1

The use of an equivalence sign in this argument does not imply exact equality. A tree contains branches, shoots and twigs of different lengths and dimensions. Consequently, each class of element possesses a range of natural frequencies. The first branch modes, for example, form a frequency band rather than a single value. The whole-tree resonances are themselves broadened by damping. Aeroelastic similarity therefore concerns the overlap or merging of modal regimes belonging to different levels of the structure.

This distinction is important. Biological variation, rather than destroying the proposed relationship, can provide a finite bandwidth over which the modal systems interact.

4 Mechanical coupling and damping

The spectral observations were accompanied by direct observations of the way motion passed through the tree. Short, high-speed gusts tended to excite twigs, shoots and some branches, whereas longer gusts displaced a larger fraction of the canopy and were followed by damped harmonic motion of the tree. The character of the response therefore depended on the time scale, and hence the frequency content, of the atmospheric excitation.

Mechanical coupling between branches could also be demonstrated directly. When one branch was manually displaced, torsional motion of the bole excited an adjacent branch. The two branches could move approximately out of phase, with the amplitude of one increasing as that of the other decreased. The exchange then reversed while the total motion progressively decayed. Similarly, a transient disturbance applied to a branch could produce visible motion of shoots and twigs elsewhere through the connected structure.

These observations indicated that energy supplied to one element need not remain confined to that element. Flexural and torsional coupling allow elastic energy to be transferred between the bole, branches and smaller canopy elements. The thesis described the resulting reduction of motion in terms of aeroelastic damping, involving aerodynamic drag together with the mechanical elastic coupling of the tree elements. In this picture the coupling redistributes energy among the elements, while damping progressively removes energy from their motion. The combined effect is a strongly damped response.

The tree was consequently interpreted as a large, well-structured shock absorber. A transient input may excite a number of higher-frequency resonances, while a longer gust excites the lower modes of the whole structure. Coupling then distributes the resulting motion among the available structural elements while damping removes energy from the system.

5 From elastic similarity to aeroelastic similarity

The mechanical design of trees had previously been considered in terms of elastic similarity (McMahon and Kronauer, 1975; McMahon, 1976). For dynamically similar elastic structures a characteristic natural frequency may be related to element length l by a scaling of the form

$$f \propto l^{-1/2}. \tag{6}$$

McMahon and Kronauer reported an experimental exponent of approximately -0.59, reasonably close to this elastic-similarity expectation.

The spruce measurements suggested, however, that the dynamic organisation of the complete tree could not be described solely by the scaling of isolated elements. The important additional feature was the interaction of the resonant modes of the different elements. The observed correspondence between whole-tree, branch, shoot and twig modes therefore led to the proposal of a qualitative extension of elastic similarity.

The term *aeroelastic similarity* is used here in its original sense: a hypothetical mechanical design principle based on a highly organised equivalence of the modes of the different tree elements. The principle is not a statement that every element has an identical frequency, nor is it simply a new frequency–length scaling law. It concerns the organisation of the complete coupled tree, including the aerodynamic environment in which its elements move.

6 The aeroelastic similarity hypothesis

The hypothesis can be stated as follows:

> *Aeroelastic similarity may be regarded as a qualitative mechanical design principle in which the number and dimensions of the structural elements of a tree are constrained such that the natural-frequency regimes of successive levels possess an organised equivalence. Through the aeroelastic interaction of the tree elements, this organisation may allow incident atmospheric energy to be partitioned among a number of coupled modes and dissipated, restricting the development of large resonant amplitudes.*

The proposed organisation is illustrated by the modal scheme in Table 1. Atmospheric frequencies corresponding to a natural-frequency regime can induce resonant motion in the associated elements. Because corresponding regimes of different elements overlap and the elements are mechanically connected, energy can pass between them. Each element then contributes to the dissipation of the incident atmospheric energy.

The effect is therefore not the suppression of resonance. Resonant responses are an essential part of the proposed mechanism. The important point is that energy is not necessarily concentrated in a single isolated resonance. Instead, it can be partitioned among a set of coupled modes distributed through the tree.

If a large quantity of energy were to remain concentrated in one mode, the amplitude might become sufficient to exceed the elastic limit of an element and cause breakage. In the proposed scheme the presence of many interacting elements provides alternative routes for the incident energy. In very strong winds individual elements may still be sacrificed by breakage, but the overall organisation may contribute to the ultimate survivability of the tree.

7 A schematic air–canopy energy interaction

The physical picture may be represented by a broad atmospheric velocity spectrum acting on a hierarchy of structural resonances. At the lowest frequencies lie the principal whole-tree modes. At progressively higher frequencies appear the modal ranges of branches, shoots and twigs. The ranges are not independent: the first branch-mode regime overlaps a higher whole-tree mode;

higher branch modes overlap shoot modes; and the hierarchy continues toward the smaller elements.

The direction of energy transfer is consequently not restricted to a single path. Wind energy can enter the structure at frequencies appropriate to the scale of the atmospheric motion and the exposed element. Mechanical coupling permits subsequent redistribution between corresponding modal regimes. Aerodynamic resistance acts throughout the moving canopy, so that the subdivision of motion among numerous elements provides a distributed means of energy dissipation.

Biological variation in the dimensions of nominally similar elements produces ranges rather than single natural frequencies. Thus branches of different sizes provide a branch-frequency bandwidth which may overlap the broadened resonance associated with the corresponding whole-tree mode. Aeroelastic similarity should therefore be interpreted in terms of overlapping modal regimes rather than exact equality of individual frequencies.

8 Mechanical design interpretation

Aeroelastic similarity was proposed as a qualitative mechanical design principle, not as a specification of the biological mechanism by which the structure is produced. The hypothesis suggests that the number and dimensions of branches, shoots, twigs and other elements may be constrained by the requirement that the resulting resonant modes form the organised pattern described above.

The proposition does not determine whether such a structure arises genetically, through mechanically active growth responses, or through some combination of these processes. Reaction wood and other biological feedback mechanisms may alter the mechanical properties of a growing tree, but the original investigation did not establish the developmental origin of the proposed modal organisation.

The mechanical argument is independent of that question. If the natural modes of successive structural levels are organised into overlapping regimes and the levels are strongly coupled, the tree possesses a means of partitioning and dissipating incident wind energy. Such a property would be advantageous in an environment dominated by intermittent and sometimes intense turbulent loading.

9 Limitations of the hypothesis

The aeroelastic similarity principle was inferred from a limited experimental investigation and was explicitly proposed as a qualitative hypothesis. Several qualifications are therefore necessary.

First, the correspondence of modes is approximate. Tree elements vary biologically in size and mechanical properties, and their resonances have finite bandwidths. Secondly, some higher modes were less securely determined than the lower modes, particularly for the smallest elements. Thirdly, the theoretical calculations treated selected elements more simply than they exist in the complete canopy and did not fully incorporate the aeroelastic interactions whose importance was indicated by the measurements. Finally, the observations established a plausible mechanical organisation but did not demonstrate that such organisation is a universal design rule for trees.

The hypothesis should therefore be understood in the form in which it arose from the measurements: as a proposed qualitative principle linking the hierarchy of tree structure to the hierarchy of its resonant motions and to the dissipation of atmospheric wind energy.

10 Conclusion

Measurements of Sitka spruce motion revealed a simple and unexpected relationship between the natural modes of successive structural elements. The first branch mode was associated with the second whole-tree mode, higher branch modes with higher whole-tree modes, and the shoot estimates suggested a continuation of the same pattern. Because nominally similar biological elements possess a range of dimensions, these correspondences are most naturally regarded as overlapping modal bands rather than exact frequency equalities.

The tree also showed strong mechanical coupling and rapid damping following transient excitation. Branch, shoot and twig motions could be excited through the connected structure, suggesting that wind-induced energy was transferred among the different elements while the motion was progressively damped. The combined observations led to the description of the tree as a large, well-structured shock absorber.

The aeroelastic similarity hypothesis proposes that this behaviour reflects a mechanical design principle: the number and dimensions of the structural elements may be constrained so that the resonant regimes of successive levels possess an approximate equivalence. Atmospheric energy can consequently be partitioned among many coupled modes rather than being concentrated in a single potentially damaging resonance. The resulting organisation may contribute to the ability of the tree to survive strong turbulent winds.

Source note

This paper is a stand-alone presentation of the aeroelastic similarity hypothesis developed in the author's PhD thesis, *Quantification of the Interactive Motions of the Atmospheric Surface Layer and a Conifer Canopy*, submitted to the Cranfield Institute of Technology in December 1983 (Scannell, 1983). The argument, observations and interpretation presented here are based on that thesis; no subsequent literature is reviewed.

References

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