

The Aeroelastic Similarity Hypothesis for Tree Structure

A Proposed Mechanical Design Principle for the Dissipation of Wind-Induced Motion

B. Scannell

Abstract

Measurements of Sitka spruce motion, made during an investigation of interactions between atmospheric turbulence and a conifer canopy, suggested an unexpected relationship between the natural-frequency modes of successive structural elements of the tree. In particular, the first branch-mode regime lay near the second whole-tree mode, while higher branch, shoot and twig modes suggested a continuing hierarchy. Sway tests and acceleration spectra provided numerical estimates of these resonances, and biological variation gave rise to ranges of natural frequencies rather than exact modal equalities. Visual observations demonstrated mechanical coupling within the tree and decay of motion following transient excitation. These results led to a qualitative hypothetical mechanical design principle termed *aeroelastic similarity*. The proposed organised equivalence of modal regimes permits incident atmospheric energy to be shared among coupled structural elements; the resulting motions are damped by aerodynamic and other dissipative effects. Such a distributed shock-absorbing response may restrict the build-up of damaging resonant amplitudes. The experimental basis, frequency evidence, proposed mode hierarchy and air-canopy energy-interaction scheme are presented here as a stand-alone account of the hypothesis developed in the author's doctoral thesis, submitted in December 1983.

1 Introduction

The response of a tree to turbulent wind is determined not only by the flexibility of its bole, but by the motions of a hierarchy of structural elements: branches, shoots, twigs and needles. Each of these elements possesses natural modes of vibration and each is mechanically connected to the other elements of the tree. The canopy is therefore a coupled aeroelastic structure exposed to a broad spectrum of atmospheric motion.

The work described here formed part of a doctoral investigation of the interactive motions of the atmospheric surface layer and a conifer canopy (Scannell, 1983). The atmospheric measurements, instrumentation and detailed turbulence results of that investigation are outside the scope of the present paper. Attention is restricted to one result arising from the measurements of tree motion: the apparent organisation of the resonant modes of the different structural levels of Sitka spruce and the mechanical interpretation that followed from it.

The original study combined natural-frequency estimates from sway tests, acceleration spectra measured during wind excitation, and theoretical modal calculations. Comparison of these results suggested a simple connection between the resonances of the separate canopy elements. Together with observations of mechanical coupling and the decay of motion following wind excitation, this led to the proposition that the tree behaves as a large, well-structured shock absorber. A qualitative mechanical design principle, termed *aeroelastic similarity*, was proposed to describe this organisation.

2 Experimental basis and frequency evidence

The original measurements were made at the RivoX plantation in Moffat Forest, southern Scotland, which contained Sitka spruce. During spring 1982 the sway periods, heights and stem diameters of 116 trees within 10 m of the instrument tower were measured. Their estimated mean first whole-tree natural frequency was 0.52 ± 0.08 Hz. The more detailed investigation employed in situ sway tests on selected trees and branches, acceleration records obtained from instruments fixed to boles or a branch under wind excitation or manual shaking, and laboratory sway tests on smaller elements (Scannell, 1983).

The laboratory specimens comprised a long branch (LB) and a short branch (SB) removed from an 8.4 m Sitka spruce, together with a shoot (S) and a twig (T) selected from the long branch. First-mode estimates for the branches were also made while they were attached to the tree. The second twig mode could not be reliably determined by the sway method. The wider field survey establishes the scale of the whole-tree observations; the detailed branch–shoot–twig measurements necessarily refer to a much smaller selection of elements.

Table 1 provides the principal numerical evidence for the proposed correspondences. Its two panels reproduce, in compact form, the estimates reported in the thesis’s Tables 37 and 47. The modes and frequency ranges are observational estimates, not fitted values chosen to make the proposed relationships exact.

Table 1: Natural-frequency evidence from the original thesis (Hz). Panel (a) summarises Table 37: unparenthesised entries are the reported sway estimates, while parenthesised ranges are additional estimates for differently sized elements. Panel (b) summarises Table 47: unparenthesised entries are spectral estimates for Tree 1 and one of its branches; parenthesised values are the sway-test estimates shown there. The two panels should not be read as measurements on one identical set of elements.

(a) *Sway-test estimates and additional ranges (thesis Table 37)*

Element	First mode	Second mode
Whole tree	0.6 (0.37–0.85)	(1.17–1.43)
Long branch (LB)	1.7 (1.25–2.4)	2.9 (2.2–3.8)
Short branch (SB)	2.4	3.8
Shoot (S)	3.0 (2.9–3.3)	4.4 (3.8–4.4)
Twig (T)	4.5 (3.9–4.5)	—

(b) *Spectral estimates for Tree 1 and a branch (thesis Table 47)*

Element	Mode 1	Mode 2	Mode 3	Mode 4
Whole tree	0.44	1.65 (1.25)	2.80	3.95
Branch ($l = 1.25$ m)	1.25 (1.34)	2.55 (2.65)	3.65	—

The frequency ranges in panel (a) illustrate, for example, an overlap between the estimated second whole-tree regime (1.17–1.43 Hz) and the range of first branch modes (1.25–2.4 Hz). The second long-branch estimate (2.9 Hz) is close to the first shoot estimate (3.0 Hz), while the second shoot and first twig estimates are 4.4 and 4.5 Hz. Panel (b) independently shows the successive, approximate correspondence of branch modes with the next higher whole-tree modes. Individual estimates can differ appreciably; the original argument concerns corresponding modal *regimes*, not numerical equality in every sample.

3 Natural modes and theoretical estimates

For the sampled elements, the experimental frequency estimates were compared with modal calculations based on measured flexibility coefficients and elemental masses. The tree elements were represented by discrete concentrated-mass systems; the influence (flexibility) coefficients were obtained by applying known loads at selected stations and measuring the resulting deflections.

For an undamped discretised element the natural modes may be written in the usual form

$$([K] - \omega^2[M]) \mathbf{q} = 0, \quad (1)$$

where $[K]$ and $[M]$ are the stiffness and mass matrices, ω is natural angular frequency and $\mathbf{q}$ is the mode shape. In the thesis the experimentally determined flexibility matrix $[C] = [K]^{-1}$ was used, giving

$$([I] - \omega^2[C][M]) \mathbf{q} = 0. \quad (2)$$

The thesis summarised the first-mode theoretical overestimates as approximately 15% for branches and approximately 30% for shoots and twigs; the discrepancy varied by element. The discrepancy was attributed principally to effects omitted when the elements were treated in relative isolation. In the actual tree, shoots, twigs and needles modify branch motion, while the bole, branches and smaller elements interact mechanically and aerodynamically. The measured motion is therefore that of an aeroelastic system rather than a collection of independent elastic cantilevers.

4 An organised equivalence of modes

Comparison of the whole-tree and branch spectra produced an unexpected result. In the modes identified for Tree 1 and its sampled branch (Table 1b), each branch mode lay near the next higher whole-tree mode. If N_i denotes the i th whole-tree mode and B_i the i th branch mode, the observed relationship may be written schematically as

$$B_1 \simeq N_2, \quad B_2 \simeq N_3, \quad B_3 \simeq N_4. \quad (3)$$

The similarity was not predicted by the isolated-element theoretical calculations. For modes that could be compared by both methods, the thesis reported agreement between sway-test and spectral estimates on average to within approximately 10%, lending support to the mode identifications.

The shoot frequencies suggested that the relationship continued to the next structural level. In particular, the first shoot mode could be associated with the second branch and third whole-tree modes, giving

$$S_1 \simeq B_2 \simeq N_3, \quad (4)$$

and the higher estimates suggested

$$S_2 \simeq B_3 \simeq N_4. \quad (5)$$

The corresponding scheme was extended to the twig scale. The full six-group modal organisation proposed in the thesis is reproduced in Table 2. Its extension beyond the directly identified modes is part of the hypothesis rather than an additional set of measured equalities.

The experimentally based correspondences are most secure for the lower modes, particularly those of the whole tree and branches. The higher entries are an extrapolation of the proposed pattern; in particular, the second twig mode was not established by the sway tests.

The use of an equivalence sign in this argument does not imply exact equality. A tree contains branches, shoots and twigs of different lengths and dimensions. Consequently, each class of element possesses a range of natural frequencies. The first branch modes, for example, form a frequency band rather than a single value. The whole-tree resonances are themselves broadened

Table 2: The complete six-group scheme of equivalent modes from thesis Table 55, written with explicit mode symbols. Each column represents a proposed correspondence of frequency regimes, not an experimentally established equality of all entries. The higher groups are a schematic extension of the identified lower modes.

Element	Group 1	Group 2	Group 3	Group 4	Group 5	Group 6
Whole tree	N_1	N_2	N_3	N_4	N_5	N_6
Branch		B_1	B_2	B_3	B_4	B_5
Shoot			S_1	S_2	S_3	S_4
Twig				T_1	T_2	T_3

by damping. Aeroelastic similarity therefore concerns the overlap or merging of modal regimes belonging to different levels of the structure.

Biological variation can therefore provide a finite bandwidth over which the modal systems may interact, rather than requiring each nominally corresponding pair of frequencies to coincide exactly.

5 Mechanical coupling and damping

The spectral observations were accompanied by direct observations of the way motion passed through the tree. Short, high-speed gusts tended to excite twigs, shoots and some branches, whereas longer gusts displaced a larger fraction of the canopy and were followed by damped harmonic motion of the tree. The character of the response therefore depended on the time scale, and hence the frequency content, of the atmospheric excitation.

Mechanical coupling between branches could also be demonstrated directly. When one branch was manually displaced, torsional motion of the bole excited an adjacent branch. The two branches could move approximately out of phase, with the amplitude of one increasing as that of the other decreased. The exchange then reversed while the total motion progressively decayed. Similarly, a transient disturbance applied to a branch could produce visible motion of shoots and twigs elsewhere through the connected structure.

These observations indicated that energy supplied to one element need not remain confined to that element. Flexural and torsional coupling allow elastic energy to be transferred between the bole, branches and smaller canopy elements. The thesis described the resulting reduction of motion in terms of aeroelastic damping, involving aerodynamic drag together with the mechanical elastic coupling of the tree elements. In this picture the coupling redistributes energy among the elements, while damping progressively removes energy from their motion. The observed motions decayed following excitation, indicating damping; no damping ratios are reported for these observations in the present paper.

The tree was consequently interpreted as a large, well-structured shock absorber. A transient input may excite a number of higher-frequency resonances, while a longer gust excites the lower modes of the whole structure. Coupling then distributes the resulting motion among the available structural elements while damping removes energy from the system.

6 From elastic similarity to aeroelastic similarity

The mechanical design of trees had previously been considered in terms of elastic similarity (McMahon and Kronauer, 1975; McMahon, 1976). For dynamically similar elastic structures a characteristic natural frequency may be related to element length l by a scaling of the form

$$f \propto l^{-1/2}. \quad (6)$$

McMahon and Kronauer reported an experimental exponent of approximately -0.59 , reasonably close to this elastic-similarity expectation.

The spruce measurements suggested, however, that the dynamic organisation of the complete tree could not be described solely by the scaling of isolated elements. The important additional feature was the interaction of the resonant modes of the different elements. The observed correspondence between whole-tree, branch, shoot and twig modes therefore led to the proposal of a qualitative extension of elastic similarity.

The term *aeroelastic similarity* is used here in its original sense: a hypothetical mechanical design principle based on a highly organised equivalence of the modes of the different tree elements. The principle is not a statement that every element has an identical frequency, nor is it simply a new frequency–length scaling law. It concerns the organisation of the complete coupled tree, including the aerodynamic environment in which its elements move.

7 The aeroelastic similarity hypothesis

The hypothesis can be stated as follows:

Aeroelastic similarity may be regarded as a qualitative mechanical design principle in which the number and dimensions of the structural elements of a tree are constrained such that the natural-frequency regimes of successive levels possess an organised equivalence. Through the aeroelastic interaction of the tree elements, this organisation may allow incident atmospheric energy to be partitioned among a number of coupled modes and dissipated, restricting the development of large resonant amplitudes.

The proposed organisation is illustrated by the modal scheme in Table 2. Atmospheric frequencies corresponding to a natural-frequency regime can induce resonant motion in the associated elements. Because corresponding regimes of different elements overlap and the elements are mechanically connected, energy can pass between them. Each element then contributes to the dissipation of the incident atmospheric energy.

The effect is therefore not the suppression of resonance. Resonant responses are an essential part of the proposed mechanism. The important point is that energy is not necessarily concentrated in a single isolated resonance. Instead, it can be partitioned among a set of coupled modes distributed through the tree.

If a large quantity of energy were to remain concentrated in one mode, the amplitude might become sufficient to exceed the elastic limit of an element and cause breakage. In the proposed scheme the presence of many interacting elements provides alternative routes for the incident energy. In very strong winds individual elements may still be sacrificed by breakage, but the overall organisation may contribute to the ultimate survivability of the tree.

8 A schematic air–canopy energy interaction

The physical picture proposed in the thesis is shown in Figure 1, redrawn from its Figure 97. A broad atmospheric velocity spectrum acts on the modal regimes of a hierarchy of structural elements. Lower-frequency disturbances predominantly excite the larger-scale motions; correspondingly higher frequencies can excite branches, shoots and twigs. The modal curves in the figure represent the conceptual correspondence of groups rather than measured spectral fits.

The conceptual correspondence between the modes permits energy entering at one frequency to be redistributed between coupled elements at nearby frequencies. Aerodynamic drag acts on moving elements throughout the canopy. Biological variation in the dimensions of nominally

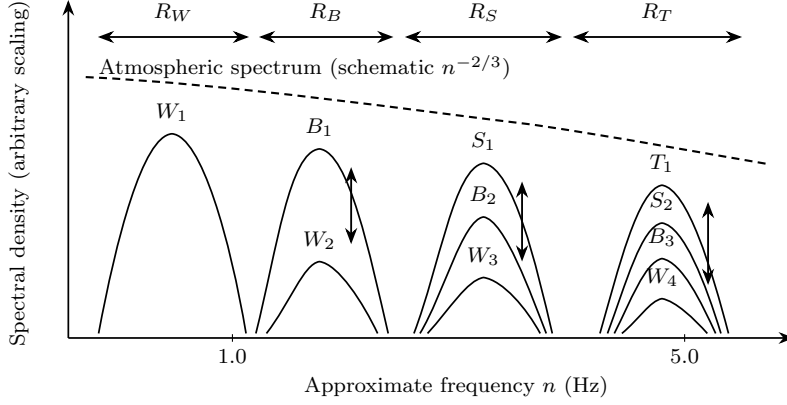


Figure 1: Schematic air-canopy aeroelastic energy interactions, redrawn from Figure 97 of the 1983 thesis. The dashed line is a conceptual atmospheric velocity spectrum; solid curves denote overlapping elemental modal regimes (W : whole tree; B : branch; S : shoot; T : twig). The double arrows indicate possible transfer between coupled resonances, and R_W , R_B , R_S and R_T illustrate modal ranges arising from biological variation. Axes and amplitudes are schematic, not a numerical fit to measurements.

similar elements provides finite modal bandwidths which may overlap the broadened resonances of elements at other structural levels. Thus the scheme does not require equality of individual natural frequencies. In the original interpretation, partition among interacting modes and subsequent damping restrict the development of exceptionally large resonant amplitudes.

9 Mechanical design interpretation

Aeroelastic similarity was proposed as a qualitative mechanical design principle, not as a specification of the biological mechanism by which the structure is produced. The hypothesis suggests that the number and dimensions of branches, shoots, twigs and other elements may be constrained by the requirement that the resulting resonant modes form the organised pattern described above.

The proposition does not determine whether such a structure arises genetically, through mechanically active growth responses, or through some combination of these processes. Reaction wood and other biological feedback mechanisms may alter the mechanical properties of a growing tree, but the original investigation did not establish the developmental origin of the proposed modal organisation.

The mechanical argument is independent of that question. If the natural modes of successive structural levels are organised into overlapping regimes and the levels are strongly coupled, the tree possesses a means of partitioning and dissipating incident wind energy. Such a property would be advantageous in an environment dominated by intermittent and sometimes intense turbulent loading.

10 Limitations of the hypothesis

The aeroelastic similarity principle was inferred from a limited experimental investigation and was explicitly proposed as a qualitative hypothesis. Several qualifications are therefore necessary.

First, the correspondence of modes is approximate. Tree elements vary biologically in size and mechanical properties, and their resonances have finite bandwidths. Secondly, some higher modes were less securely determined than the lower modes, particularly for the smallest elements.

Thirdly, the theoretical calculations treated selected elements more simply than they exist in the complete canopy and did not fully incorporate the aeroelastic interactions whose importance was indicated by the measurements. Finally, the observations established a plausible mechanical organisation but did not demonstrate that such organisation is a universal design rule for trees.

The hypothesis should therefore be understood in the form in which it arose from the measurements: as a proposed qualitative principle linking the hierarchy of tree structure to the hierarchy of its resonant motions and to the dissipation of atmospheric wind energy.

11 Conclusion

Measurements of Sitka spruce motion revealed a simple and unexpected relationship between the natural modes of successive structural elements. The first branch mode was associated with the second whole-tree mode, higher branch modes with higher whole-tree modes, and the shoot estimates suggested a continuation of the same pattern. Because nominally similar biological elements possess a range of dimensions, these correspondences are most naturally regarded as overlapping modal bands rather than exact frequency equalities.

The tree also showed mechanical coupling and observable decay of motion following transient excitation. Branch, shoot and twig motions could be excited through the connected structure, suggesting that wind-induced energy was transferred among the different elements while the motion was progressively damped. The combined observations led to the description of the tree as a large, well-structured shock absorber.

The aeroelastic similarity hypothesis proposes that this behaviour reflects a mechanical design principle: the number and dimensions of the structural elements may be constrained so that the resonant regimes of successive levels possess an approximate equivalence. Atmospheric energy may consequently be partitioned among many coupled modes rather than being concentrated in a single potentially damaging resonance. The resulting organisation may contribute to the ability of the tree to survive strong turbulent winds.

Source note

This paper is a stand-alone presentation of the aeroelastic similarity hypothesis developed in the author's PhD thesis, *Quantification of the Interactive Motions of the Atmospheric Surface Layer and a Conifer Canopy*, submitted to the Cranfield Institute of Technology in December 1983 (Scannell, 1983). The argument, observations and interpretation presented here are based on that thesis; no subsequent literature is reviewed.

References

- T. A. McMahon. The mechanical design of trees. *Scientific American*, 233:92–102, 1976.
- T. A. McMahon and R. E. Kronauer. Tree structures: Deducing the principle of mechanical design. *Journal of Theoretical Biology*, 59:443–446, 1975.
- B. Scannell. *Quantification of the Interactive Motions of the Atmospheric Surface Layer and a Conifer Canopy*. PhD thesis, Cranfield Institute of Technology, Ecological Physics Research Group, December 1983.